Quantum Gauge Invariance and Grand Unification

Promotionskolloquium

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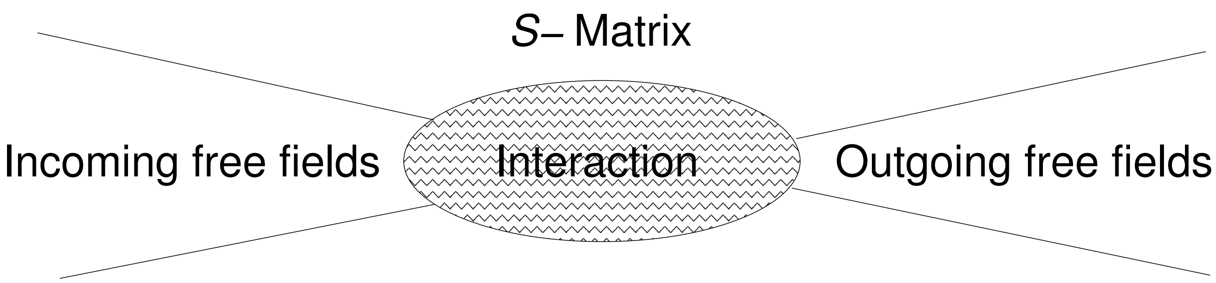
Contents

What is the causal approach?

The central object in our approach is the S-Matrix:



$$S(g) = \mathbb{1} + \sum_{n=1}^{\infty} \frac{1}{n!} \int d^4x_1 \cdots d^4x_n T_n(x_1, \ldots, x_n) g(x_1) \cdots g(x_n)$$

T_n: operator valued distribution, chronological products; expressed in terms of *free* fields only

g: tempered test function that switches the interaction on and off; adiabatic limit: $g \longrightarrow 1$ carried out in the end

Causality for the S-Matrix means:

$$S(g_1 + g_2) = S(g_2)S(g_1)$$

when g_1, g_2 have disjoint supports in Minkowski space $\mathbb{M}$

$$\operatorname{supp} g_1 \subset \{x \in \mathbb{M} | x^0 \in (-\infty, r)\},$$
$$\operatorname{supp} g_2 \subset \{x \in \mathbb{M} | x^0 \in (r, +\infty)\}.$$

Handed down to the T's:

$$T_n(x_1, \ldots, x_n) = T_m(x_1, \ldots, x_m)T_{n-m}(x_{m+1}, \ldots, x_n)$$

if

$$\{x_{m+1}, \ldots, x_n\} \text{ are earlier than } \{x_1, \ldots, x_m\}$$

Some Remarks on the S-Matrix

1. S is a unitary operator on the physical subspace $\mathcal{F}_{\text{phys}}$

$$\lim_{g \to 1} \mathsf{P} S(g)^{+} \mathsf{P} S(g) \mathsf{P} = \mathsf{P}$$

   P: Projection operator on $\mathcal{F}_{\text{phys}}$

2. Infrared divergences:
   – massless fields only: Strong adiabatic limit does not exist
   – massive fields only: Limit exists
   – massless and massive fields: Interesting!

3. Ultraviolet divergences: Time ordering T of the T_n: $T_n(x_1, \ldots, x_n) = \mathsf{T}[T_1(x_1) \cdots T_1(x_n)]$ with causality and distribution splitting, no cut-off needed

Foundation and development of the causal approach

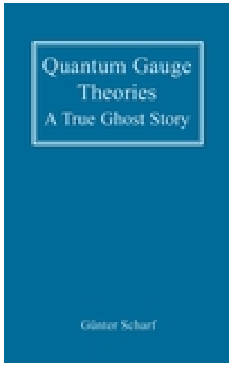
– E. C. G. Stückelberg (construction of time-ordered products)

– N. N. Bogoliubov and D. V. Shirkov (further developed in *Introduction to the theory of quantized fields.* Wiley-Interscience, New York 1959.)

– H. Epstein and V. Glaser: *The Role of Locality in Perturbation Theory.* Annales Poincaré Phys. Theor. A **19** (1973) 211.

– G. Scharf: *Finite Quantum Electrodynamics. The Causal Approach*. Springer, Berlin 1995, 2.ed.

– G. Scharf: *Quantum Gauge Theories — A True Ghost Story*. Wiley-Interscience, New York 2001.



Quantum Gauge Variation

The field operators are varied like

$$A'^{\mu}(x) = A^{\mu}(x) + \underline{\lambda\partial^{\mu}u(x)} + \mathcal{O}(\lambda^2)$$

$u(x)$: a free quantum field, fulfils the wave equation

$$\Box u(x) \overset{!}{=} 0$$

A' must have the same commutation relations as A

$$A'^{\mu}(x) = e^{-i\lambda Q} A^{\mu}(x) e^{i\lambda Q}$$

Q: gauge charge. Lie series expansion gives:

$$A'^{\mu}(x) = A^{\mu}(x) - \underline{i\lambda[Q, A^{\mu}(x)]} + \mathcal{O}(\lambda^2)$$

$$[Q, A^{\mu}(x)] = i\partial^{\mu}u(x)$$

The gauge charge Q

Q (proposed by Kugo and Ojima (1979)) in the massless case

$$Q = \int d^3x \, \partial_\nu A^\nu \overleftrightarrow{\partial_0} \, u$$

$\leftrightarrow$: derivative ∂_0 acts on both sides.

Massive case:

$$Q = \int d^3x \, (\partial_\nu A^\nu + m\Phi) \overleftrightarrow{\partial_0} \, u$$

Φ : bosonic ghost field, scalar partner for the massive vector field A^μ; Q is nilpotent

$$\boxed{Q^2 = 0} \iff u \text{ quantised as a Fermi field.}$$

T: Wick monomial of Bose fields + *even* number of Fermi fields (ghosts):

$$[Q, T] =: d_Q$$

T: Wick monomial of Bose fields + *odd* number of Fermi fields:

$$\{Q, T\} =: d_Q$$

i.e. d_Q acts like a graded derivation. Nilpotency of Q implies

$$d_Q^2 = 0$$

making a link to homological algebra. The physical subspace $\mathcal{F}_{\text{phys}}$ is

$$\boxed{\mathcal{F}_{\text{phys}} = \text{Kernel}\, Q / \overline{\text{Range}\, Q}}\,.$$

The gauge variation d_Q and gauge invariance

A theory is called <u>gauge invariant to first order</u> if

$$d_Q T_1 = \underbrace{i\partial_\mu T^\mu_{1/1}}_{\text{divergence form}}$$

$T^\mu_{1/1}$ is called Q-vertex.

Formal generalisation to higher orders:

$$d_Q T_n = i \sum_{l=1}^{n} \frac{\partial}{\partial x_l^\mu} T^\mu_{n/l}(x_1, \ldots, x_n)$$

$T^\mu_{n/l}$: Q-vertex of order n at place x_l

Second order gauge invariance

In second order the problem of anomalies arises:

According to the causal construction one has to calculate the distribution

$$D_2(x,y) = T_1(x)T_1(y) - T_1(y)T_1(x)$$

that has a causal support $\mathrm{supp} \subset \{(x-y)^2 \geq 0\}$; it must be decomposed into a retarded and an advanced part

$$\mathrm{supp} R_2 \subset \overline{V}^+, \ \mathrm{supp} A_2 \subset \overline{V}^- .$$

The splitting is not unique, if the considered diagram is of singular order $\omega \geq 0$! $\longrightarrow$ Normalisation terms.

Question: Is gauge invariance preserved in the distribution splitting?

– D_2 itself is gauge invariant

– R_2 agrees with D_2 on the forward light cone $V^+/\{0\}$

– similarly for $R^\mu_{2/1}, R^\mu_{2/2}$

That means: gauge invariance can only be *violated by local terms (anomalies)* in the following way:

If $T^\mu_{1/1}$ contains terms with a derivative ∂^μ then the commutator is proportional to

$\partial^\mu_x D_m(x-y) \overset{\text{splitting}}{\longrightarrow} \partial^\mu_x D^{\text{ret}}_m(x-y)$, D_m : Jordan-Pauli distribution.

Applying the derivative ∂^μ_x from the definition gives

$$\Box_x D^{\text{ret}}_m(x-y) = -m^2 D^{\text{ret}}_m + \delta(x-y)$$

leading to an anomaly.

The local terms are exactly the normalisation terms from the splitting: $N_2, N^\mu_{2/1}, N^\mu_{2/2}$, giving rise to the definition:

The theory is gauge invariant to second order if

$$d_Q(R_2 + N_2) = \partial^x_\mu(R^\mu_{2/1} + N^\mu_{2/1}) + \partial^y_\mu(R^\mu_{2/2} + N^\mu_{2/2})$$

$\rightarrow T_2 = T_2(R_2, N_2, R'_2)$ is gauge invariant, with $R'_2(x, y) = -T_1(x)T_1(y)$

Summary: Commuting the factors with derivative ∂^μ in $T^\mu_{1/1}$ with all terms $T_1(y)$ gives tree-graph contributions with four external legs (sectors).

Gauge Invariance for a general massive gauge theory

Free asymptotic fields

– r massive and s massless gauge fields A_a^μ, $a = 1, \ldots, r+s$

– $r+s$ fermionic ghosts u_a and anti-ghosts $\tilde{u}_a$ of mass m_a, $m_a = 0$ for $a > r$

– scalar partners $\Phi_a(x)$, $a \leq r$ of the same mass m_a:

$$\begin{aligned}
[A_a^\mu(x), A_b^\nu(y)] &= i\delta_{ab} g^{\mu\nu} D_{m_a}(x-y) \\
\{u_a(x), \tilde{u}_b(y)\} &= i\delta_{ab} D_{m_a}(x-y) \\
[\Phi_a(x), \Phi_b(y)] &= i\delta_{ab} D_{m_a}(x-y),
\end{aligned}$$

$$D_{m_a}(x) = \frac{i}{(2\pi)^3} \int d^4p\, \delta(p^2 - m_a^2) \mathrm{sgn}(p^0) e^{-ixp}$$

The couplings are the following normally orders products of free fields

$$T_1^0 = igf_{abc}(A_{\mu a}A_{\nu b}\partial^\nu A_c^\mu - A_{\mu a}u_b\partial^\mu \tilde{u}_c)$$
$$T_1^1 = igf_{ahj}^1 A_a^\mu(\Phi_h\partial_\mu\Phi_j - \Phi_j\partial_\mu\Phi_h),\ f_{ahj}^1 = -f_{ajh}^1$$
$$T_1^2 = igf_{abh}^2 A_{\mu a}A_b^\mu\Phi_h,\ f_{abh}^2 = f_{bah}^2$$
$$T_1^3 = igf_{abh}^3 \tilde{u}_a u_b \Phi_h$$
$$T_1^4 = igf_{hjk}^4 \Phi_h\Phi_j\Phi_k,\ f_{hjk}^4\text{: symmetric in } h, j, k$$

g: coupling constant

f_{abc}: totally antisymmetric Yang-Mills structure constants

Higgs couplings: Replacement of the scalar Φ's by Higgs fields φ's

$$
\begin{aligned}
T_1^5 &= igf^5_{ahp}A^\mu_a(\Phi_h\partial_\mu\varphi_p - \varphi_p\partial_\mu\Phi_h) \\
T_1^6 &= igf^6_{apq}A^\mu_a(\varphi_p\partial_\mu\varphi_q - \varphi_q\partial_\mu\varphi_p), \quad f^6_{apq} = -f^6_{aqp} \\
T_1^7 &= igf^7_{abp}A_{\mu a}A^\mu_b\varphi_p, \quad f^7_{abp} = f^7_{bap} \\
T_1^8 &= igf^8_{abp}\tilde{u}_a u_b\varphi_p \\
T_1^9 &= igf^9_{hjp}\Phi_h\Phi_j\varphi_p, \quad f^9_{hjp} = f^9_{jhp} \\
T_1^{10} &= igf^{10}_{hpq}\Phi_h\varphi_p\varphi_q, \quad f^{10}_{hpq} = f^{10}_{hqp} \\
T_1^{11} &= igf^{11}_{pqu}\varphi_p\varphi_q\varphi_u.
\end{aligned}
$$

First order gauge invariance: couplings $f^1, \ldots, f^{10}$ are related to the Yang-Mills parameters f_{abc}

Second order: First Jacobi identity from the sector $uA\tilde{u}u$

$$
f_{abc}f_{cef} + f_{eac}f_{cbf} + f_{bec}f_{caf} = 0
$$

Second: normalisation terms and non-trivial relations between the f_{abc}'s and the gauge boson masses, e.g. from the sector $uA\Phi\Phi$:

$$\begin{aligned}
&\sum_{p=1}^{t} f^5_{ajp} f^5_{dhp} - f^5_{ahp} f^5_{djp} \\
&= \sum_{c=1}^{r+s} \frac{m_j^2 + m_h^2 - m_c^2}{2 m_h m_j} f_{dac} f_{chj} \\
&\quad - \sum_{k=1}^{r} \frac{m_k^2 + m_j^2 - m_a^2}{m_j m_k} \frac{m_k^2 + m_h^2 - m_d^2}{4 m_h m_k} f_{ajk} f_{dhk} \\
&\quad + \sum_{k=1}^{r} \frac{m_k^2 + m_h^2 - m_a^2}{m_h m_k} \frac{m_k^2 + m_j^2 - m_d^2}{4 m_j m_k} f_{ahk} f_{djk}.
\end{aligned}$$

Third order: Higgs couplings fully determined for *one* Higgs, general relation otherwise.

Testing $SU(5)$

Why $SU(5)$?

Experimentally:

$$\tau_{\text{proton}} > 2.1 \times 10^{29}\,\text{yrs (mode independent)}$$

(http://pdg.lbl.gov/2004/)

Mode dependent: $\tau_{\text{proton}} > 10^{31} - 10^{33}\,\text{yrs}$, e.g.: $\tau(p \to \pi^0 e^+) > 5.5 \times 10^{32}\,\text{yrs}$,

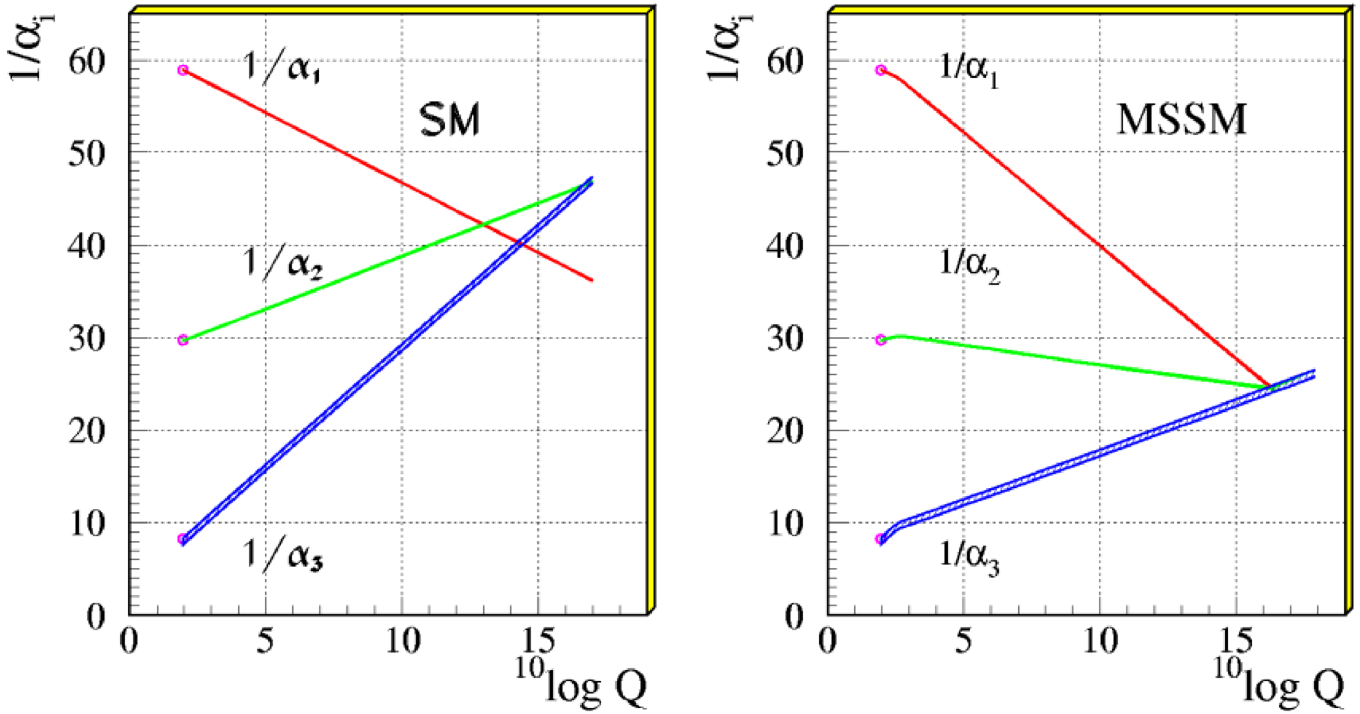
whereas in $SU(5)$ proton lifetime is

$$\tau(p) \approx \frac{{m_X}^4}{g^2_{SU(5)}{m_p}^5} = 10^{27} \div 10^{31} \;\; \text{yrs},$$

m_X: unification mass, $g_{SU(5)}$: $SU(5)$ coupling constant, m_p: proton mass

Running coupling constants for $SU(5)$

Coupling constants do not meet in simple $SU(5)$



$\alpha_{1,2,3}$: electromagnetic, weak and strong coupling constants

Wim de Boer: Phys. Lett. B **585** (2004) 276.

Breaking patterns

$SU(5)$ is nevertheless interesting because

$$
\begin{array}{l}
SO(10) \longrightarrow SU(5) \times U(1) \longrightarrow SU(5) \\
\hphantom{SO(10) \longrightarrow SU(5) \times U(1) \longrightarrow\ } \downarrow m_X \\
\hphantom{SO(10) \longrightarrow SU(5) \times U(1) \longrightarrow\ } SU(3) \times SU(2) \times U(1) \\
\hphantom{SO(10) \longrightarrow SU(5) \times U(1) \longrightarrow\ } \downarrow m_W, m_Z \\
\hphantom{SO(10) \longrightarrow SU(5) \times U(1) \longrightarrow\ } SU(3) \times U(1);
\end{array}
$$

one other breaking scheme without $SU(5)$ is

$$
\begin{aligned}
SO(10) &\longrightarrow SU(4) \times SU(2) \times SU(2) \\
&\longrightarrow \ldots \\
&\longrightarrow SU(3) \times SU(2) \times U(1) \\
&\longrightarrow SU(3) \times U(1).
\end{aligned}
$$

Masses of the $SU(5)$ gauge bosons

For a $\underline{24}$ and a $\underline{5}$ we have the Higgs potential (Buras et al., 1978)

$$V(H,\Phi) = -\frac{1}{2}\mu^2 \operatorname{tr}\Phi^2 + \frac{1}{4}\kappa(\operatorname{tr}\Phi^2)^2 + \frac{1}{2}\rho \operatorname{tr}\Phi^4 - \frac{1}{2}\nu^2 H^\dagger H + \frac{1}{4}\lambda(H^\dagger H)^2 + \alpha H^\dagger H \operatorname{tr}\Phi^2 + \beta H^\dagger \Phi^2 H$$

H: Higgs multiplet from $\underline{5}$, Φ from $\underline{24}$

$\mu, \kappa, \rho, \lambda, \alpha, \beta$: coupling constants

With the VEV's of H and Φ one calculates the gauge boson masses:

$$m_W \neq m_Z \ll m_Y \neq m_X, \quad m_\gamma = m_{\text{gluons}} = 0$$

i.e., apart from the weak, two new masses (m_X, m_Y) .

Charges of the $SU(5)$ gauge bosons

Electric charge Q is an additive quantum number $\longrightarrow$ linear combination of the diagonal generators T_0, T_3 in $SU(5)$ like "isospin"

$$Q = T_3 + cT_0, \quad c = -(5/3)^{1/2}$$

implying the explicit form for the charge operator

$$Q = \mathrm{diag}(-1/3, -1/3, -1/3, 1, 0), \quad \mathrm{tr}\, Q = 0.$$

From the general transformation property in $SU(5)$ one calculates the charges for the gauge bosons as

$$Q(W^{\pm}) = \pm 1,\ Q(\gamma, \lambda) = 0,$$
$$Q(X^{\pm}) = \pm 4/3,\ Q(Y^{\pm}) = \pm 1/3.$$

Structure constants for $SU(5)$. Three steps

1. Calculation of <u>concrete values</u> for the structure constants

Problem: Freedom for rotations among the diagonal (massless) generators and freedom for rotations among the generators of the same mass

2. Restricted coupling structure by <u>gauge invariance</u> and <u>charge conservation</u>:

Massless/Massive:	$f_{\lambda\lambda\lambda}, f_{YY\lambda}, f_{XX\lambda}, f_{YY\gamma}, f_{XX\gamma}, f_{WW\gamma}$
Purely massive couplings:	$f_{WWZ}, f_{XXZ}, f_{YYZ}, f_{XYW}$

λ: gluons, W: $W_{1,2}$, $X \in \{X_1, \ldots, X_6\}$, $Y \in \{Y_1, \ldots, Y_6\}$

3. Sums of products of structure constants as <u>free linear parameters</u> with $f_{W_1,W_2,Z} \neq 0$

Procedure for the three steps

1. A general rotation contains too many free parameters, several special rotations checked.

2. All restrictions from different sectors considered separately.
Example: From the sector $uA\Phi\Phi$ we get, setting $d = X^1$, $a = W^1$, $h = Z$, $j = Y^1$ with the restricted coupling structure

$$\sum_{c=1}^{r+s} f_{dac} f_{hjc} \rightarrow \sum_{c=1}^{r+s} f_{X^1W^1c} f_{ZY^1c} \rightarrow -\sum_{j=1}^{6} f_{X^1Y^jW^1} f_{Y^1Y^jZ}$$

$$\sum_{k=1}^{r} f_{ajk} f_{dhk} \rightarrow \sum_{k=1}^{r} f_{W^1Y^1k} f_{X^1Zk} \rightarrow +\sum_{i=1}^{6} f_{X^iY^1W^1} f_{X^1X^iZ}$$

$$\sum_{k=1}^{r} f_{ahk} f_{djk} \rightarrow \sum_{k=1}^{r} f_{W^1Zk} f_{X^1Y^1k} \rightarrow -f_{W^1W^2Z} f_{X^1Y^1W^2}$$

$$0 = \sum_{j=1}^{6} f_{X^1 Y^j W^1} f_{Y^1 Y^j Z} + \frac{m_Y^2 + m_X^2 - m_W^2}{2m_X^2} \sum_{i=1}^{6} f_{X^i Y^1 W^1} f_{X^1 X^i Z}$$
$$+ \frac{m_Y^2 + m_W^2 - m_X^2}{2m_W^2} f_{W^1 W^2 Z} f_{X^1 Y^1 W^2}$$

$\Longleftrightarrow$

$$0 = 2B m_X^2 m_W^2 + A(m_Y^2 + m_X^2 - m_W^2) m_W^2 + C(m_Y^2 + m_W^2 - m_X^2) m_X^2$$

$+$ 6 other similar equations for other choices of the indices

$$\left| \begin{array}{l} 0 = 2Bm_X^2 m_W^2 + A(m_Y^2 + m_X^2 - m_W^2)m_W^2 + C(m_Y^2 + m_W^2 - m_X^2)m_X^2 \\ 0 = 2Am_Y^2 m_W^2 + B(m_Y^2 + m_X^2 - m_W^2)m_W^2 C(m_W^2 + m_X^2 - m_Y^2)m_Y^2 \\ 0 = 2Cm_X^2 m_Y^2 + A(m_X^2 + m_W^2 - m_Y^2)m_Y^2 + B(m_Y^2 + m_W^2 - m_X^2)m_X^2 \\ 0 = 2Cm_Y^2 m_X^2 + A(2m_X^2 - m_Z^2)m_Y^2 + B(2m_Y^2 - m_Z^2)m_X^2 \\ 0 = 2Bm_X^2 m_W^2 + A(2m_X^2 - m_Z^2)m_W^2 + C(2m_W^2 - m_Z^2)m_X^2 \\ 0 = 2Am_Y^2 m_W^2 + C(2m_W^2 - m_Z^2)m_Y^2 + B(2m_Y^2 - m_Z^2)m_W^2 \end{array} \right| ;$$

similar sets of equations arise from the remaining sectors.

3. Three bigger sets of equations:

Four different indices: $\sum_c f_{12c} f_{34c}$, $1 \neq 2 \neq 3 \neq 4$:

Linear system of rank 6

$$\left.\begin{array}{ll} m_1 \neq m_2 & m_3 \neq m_4 \\ m_1 \neq m_2 & m_3 = m_4 \\ m_1 = m_2 & m_3 \neq m_4 \end{array}\right\} \text{First solution}$$

Linear system of rank 5

$$m_1 = m_2 \quad m_3 = m_4 \ \text{Second solution.}$$

Three different indices: $\sum_c f_{12c} f_{24c}$, $1 \neq 2 \neq 4$: set of equations of rank 3 (rank 2 for massive couplings only).

Two different indices: $\sum_c (f_{12c})^2$, $1 \neq 2$: Elimination of the Higgs-couplings gives

$$(m_1^2 - m_2^2) \sum_{d=\text{massless}} f_{12d}^2 = 0$$

Example for a solution from the case of four different indices

Same input of indices as above: $f_{ZW^15}f_{X^1Y^15} =: f[5]$, $5 \equiv W^2$ which leads to the following other products

$$\begin{aligned}
g[6] &:= f_{ZX^16}f_{W^1Y^16} \\
h[7] &:= f_{ZY^17}f_{W^1X^17} \\
\rightarrow h[8] &:= f_{ZY^18}f_{W^1X^18} \\
f[9] &:= f_{ZX^19}f_{W^1Y^19} \\
g[9] &:= f_{ZX^19}f_{W^1Y^19} \\
h[9] &:= f_{ZY^19}f_{W^1X^19}
\end{aligned}$$

One specific solution comes with

$$\begin{aligned}
m[1] &= m_Z^2 \\
m[2] &= m[5] = m_W^2 \\
m[3] &= m[6] = m[8] = m_X^2 \\
m[4] &= m[7] = m_Y^2,
\end{aligned}$$

(9: massless index)

i.e. 7 products for the rank 6 system. Setting $h[7] \equiv 1$ gives for the first product:

$$
\begin{aligned}
& f_{ZW^1W^2} f_{X^1Y^1W^2} \\
&= (m_X^2 m_W^4 m_Z^2 - m_W^4 m_Y^2 m_Z^2 + m_W^4 m_Y^4 - m_X^2 m_Y^2 m_W^4 \\
&\quad + m_Y^4 m_W^2 m_Z^2 - m_X^2 m_Y^2 m_Z^2 m_W^2 + 3 m_X^4 m_Y^2 m_W^2 - m_W^2 m_Y^6 \\
&\quad - 2 m_X^6 m_W^2 + 2 m_X^4 m_Y^2 m_Z^2 - m_X^2 m_Y^4 m_Z^2 - m_X^6 m_Z^2 - 2 m_Y^4 m_X^4 \\
&\quad + 2 m_X^8 - 2 m_X^6 m_Y^2 + 2 m_Y^6 m_X^2) m_W^2 \Big/ \\
&\quad \big(m_Z^2 (m_W^6 - m_X^2 m_W^4 - m_X^4 m_W^2 + m_X^6 - m_Y^2 m_W^4 + 2 m_X^2 m_Y^2 m_W^2 \\
&\quad - m_X^4 m_Y^2 - m_Y^4 m_W^2 - m_X^2 m_Y^4 + m_Y^6) m_X^2 \big) \\
&= \mathcal{O}(1) \\
&\quad (\text{if } m_{X,Y} \gg m_{Z,W} \text{ and } m_Y \approx m_X \pm \mathcal{O}(m_W)).
\end{aligned}
$$

For $g[6], h[8]$ one gets similar polynomials, though of different order of magnitude: $g[6] = h[8] = \mathcal{O}(\frac{m_Y^2}{m_W^2})$, $f[9], g[9], h[9]$ are zero.

There is also a solution here setting $m_X = m_Y$.

Results

1. With conventional structure constants we cannot achieve gauge invariance. Special rotations do not help.

2. There is also no solution for the restricted coupling structure.

3. Solutions for different mass degeneracies!

- Sets of equations of low rank, coupling structure cannot be fully derived

- Orders of magnitude for the products of structure constants: not so suggestive though not conclusive.

- $SU(5)$-compatible or favourable for an other breaking scheme?

The role of symmetries

> "[...] does the notion of symmetry, so prevalent in many ideas for probing Nature's secrets, really have the fundamental role that it is often assumed to have? I do not see why this need always be so. It does not necessarily strike me that basing particle physics on some large symmetry group (which is part of the GUT philosophy) is really a 'simple' picture, as far as a fundamental physical theory is concerned. To me, large geometrical symmetry groups are complicated rather than simple things. It might well be the case that there are fundamental asymmetries inherent in nature's laws, and that the symmetries that we see are often merely approximate features that do not persist right down to the deepest levels."

Roger Penrose: The Road to Reality. Jonathan Cape, London 2004, §28.3.

Addendum: Allowing for non-standard couplings

So far: Check for all combinations for the additional couplings f_{ZXY}, f_{WXX}, f_{WYY} in the relations.

Allowing for more couplings e.g.:

$$
\begin{aligned}
f[5 = W] &= f_{ZW^1W^2} f_{X^1X^2W^2} \\
g[6 = X] &= \sum_i f_{ZX^1X^i} f_{W^1X^2X^i} \\
h[7 = W] &= f_{ZX^2W^2} f_{W^1X^1W^2} \\
h[8 = X] &= \sum_i f_{ZX^2X^i} f_{W^1X^1X^i} \\
f[9 = \lambda_i, \gamma] &= f_{ZX^19} f_{W^1X^29} \\
g[9] &= f_{ZX^19} f_{W^1X^29} \\
h[9] &= f_{ZX^29} f_{W^1X^19}.
\end{aligned}
$$

Gauge invariance is not strong enough to determine all the couplings! Some special choices considered for a theory with three and four masses.